11.1 Empirical Rule

There are two commonly-accepted ways to do this …

Option: **Take the mean**

$$Q_1 = \frac{61 + 63}{2} = 62$$

$$Q_3 = \frac{75 + 78}{2} = 76.5$$

Option: **Choose the bigger one**

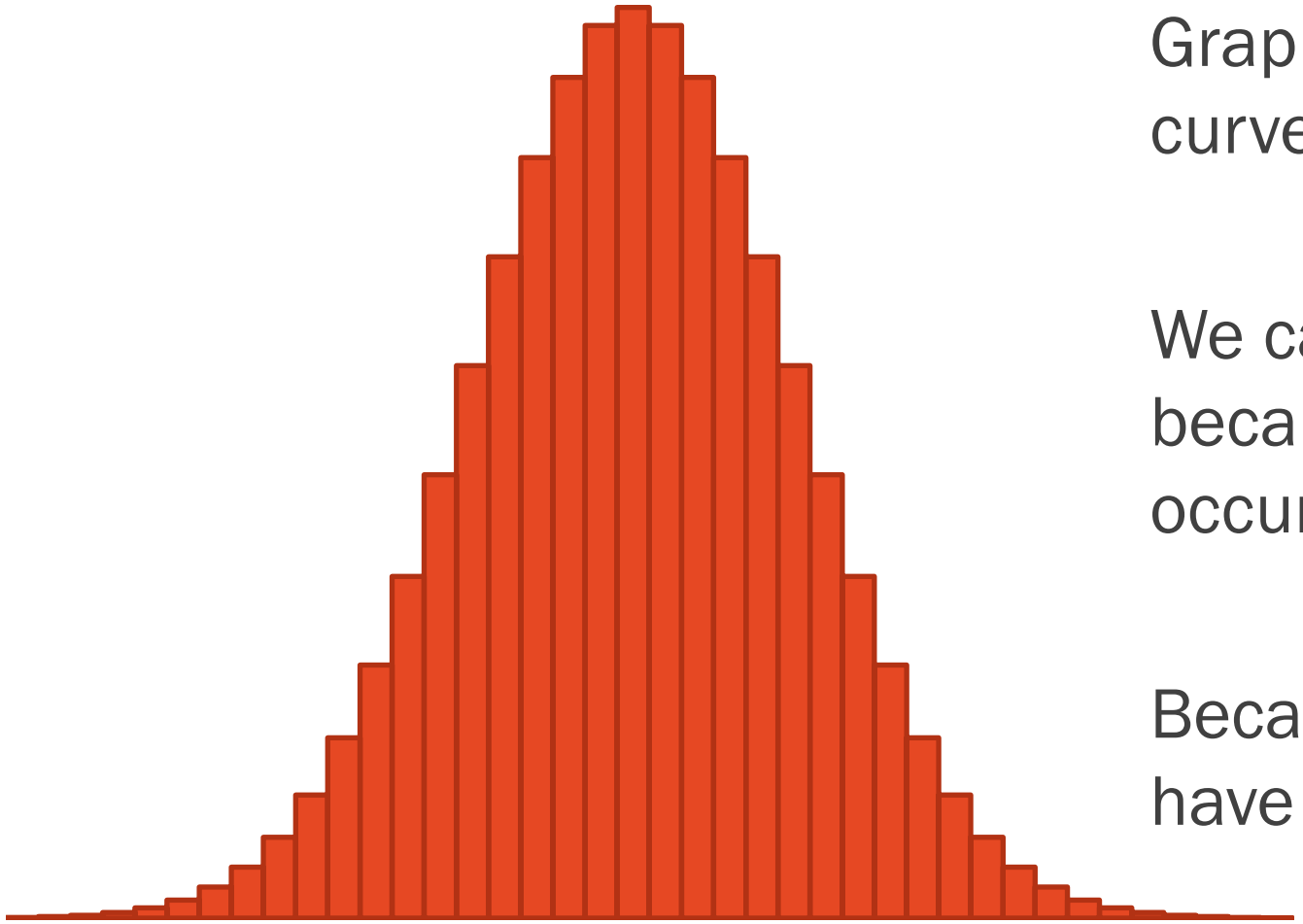
$$Q_1 = 63$$

$$Q_3 = 78$$

What you will learn

- Calculate probabilities using normal distributions and the Empirical Rule
- Recognize data sets that are normal

Normal Distributions



Graph is a **symmetric, bell-shaped** curve called the **normal curve.**

We call the distribution “normal” because of how commonly it occurs!

Because it’s so common, people have studied it very closely.

What makes a normal distribution “normal?”

This is something mathematicians are still working to fully understand!

We have noticed a mathematical fact:

This fact is called the “Central Limit Theorem”

If you measure multiple things, each with a different distribution …

… and then **add the data together** (or, averaging the data works too) …

… the sum (or average) will be a **normal distribution!**

What to Expect when Throwing Dice and Adding Them Up | by Juan Luis Ruiz-Tagle | Cantor’s Paradise

What makes a normal distribution “normal?”

This principle works **backwards** too: if the thing you’re measuring is the sum of several factors, each factor with its own distribution, then you will measure a normal distribution!

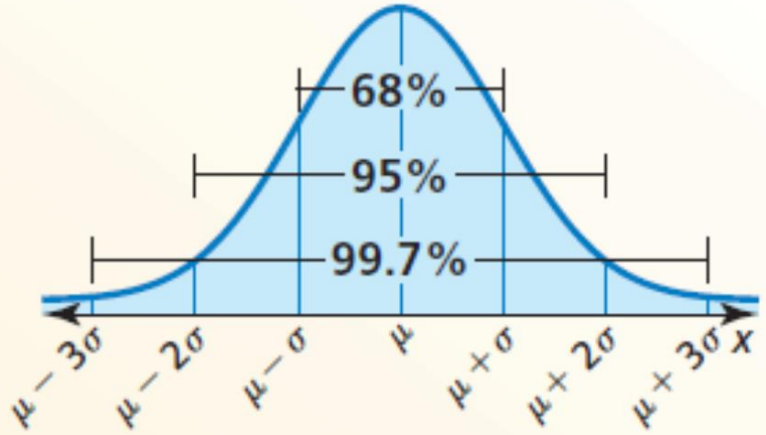
Example: What are some factors that affect how tall somebody is?

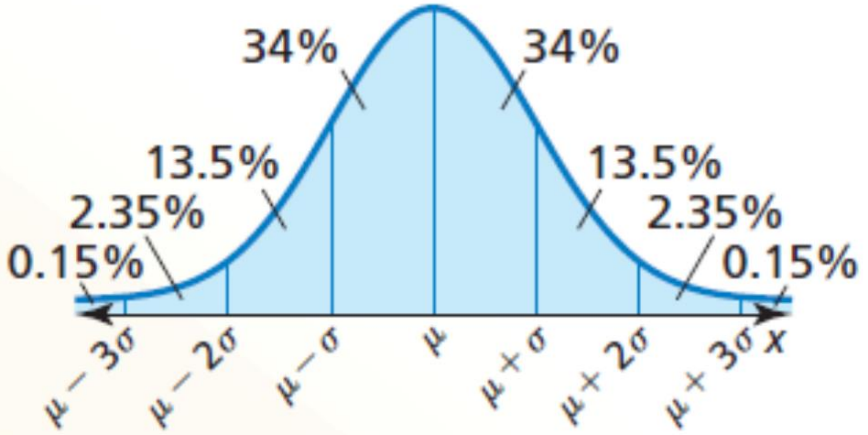
Core Concept

Areas Under a Normal Curve

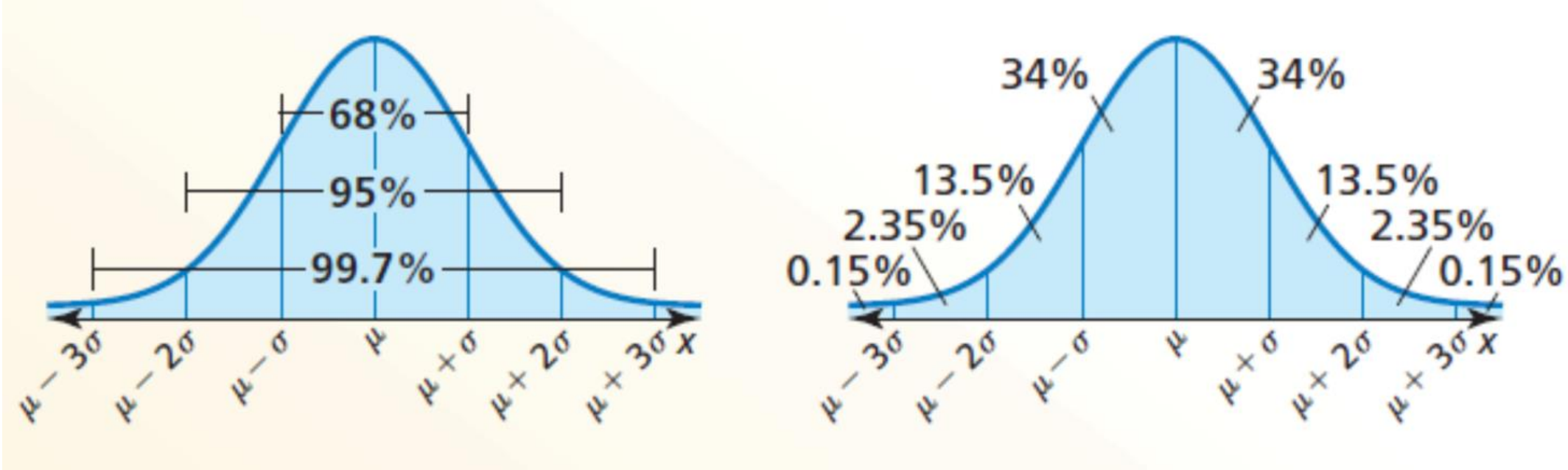
A normal distribution with mean μ and standard deviation σ has these properties.

- The total area under the related normal curve is 1.
- About 68% of the area lies within 1 standard deviation of the mean.
- About 95% of the area lies within 2 standard deviations of the mean.
- About 99.7% of the area lies within 3 standard deviations of the mean.



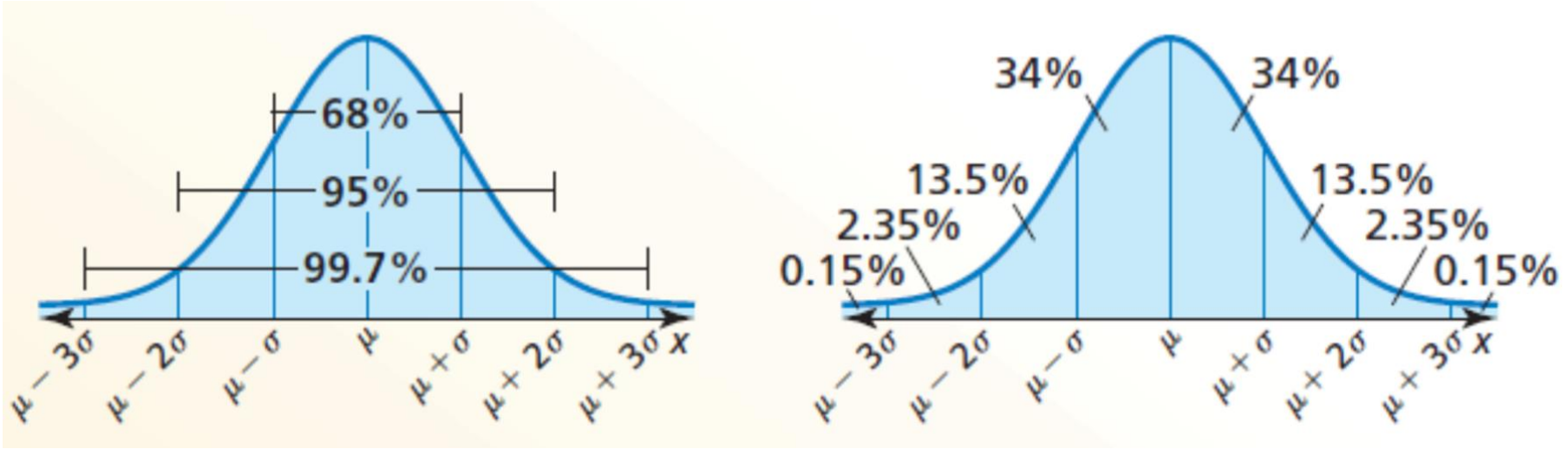


A normal distribution has mean μ and standard deviation σ. An x-value is randomly selected from the distribution. Find $P(\mu - 2\sigma \leq x \leq \mu)$.

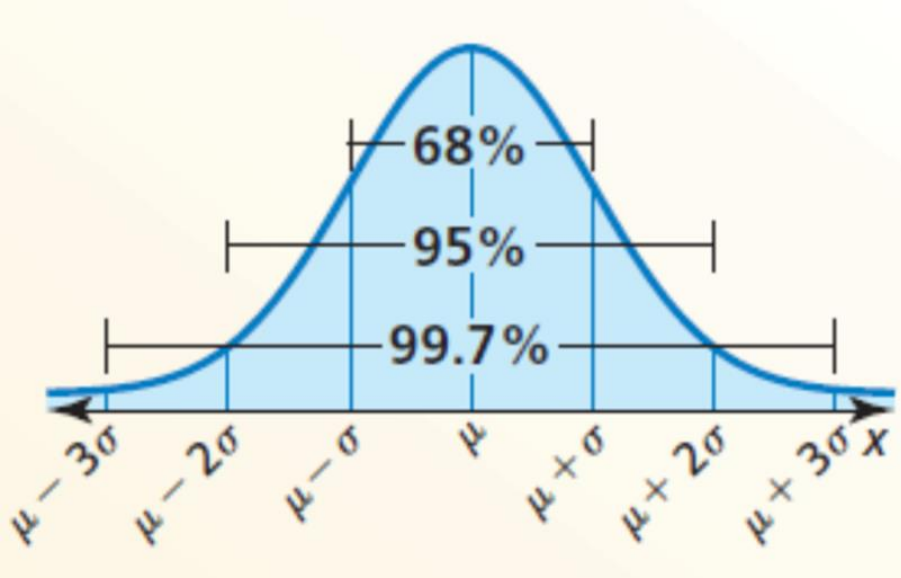


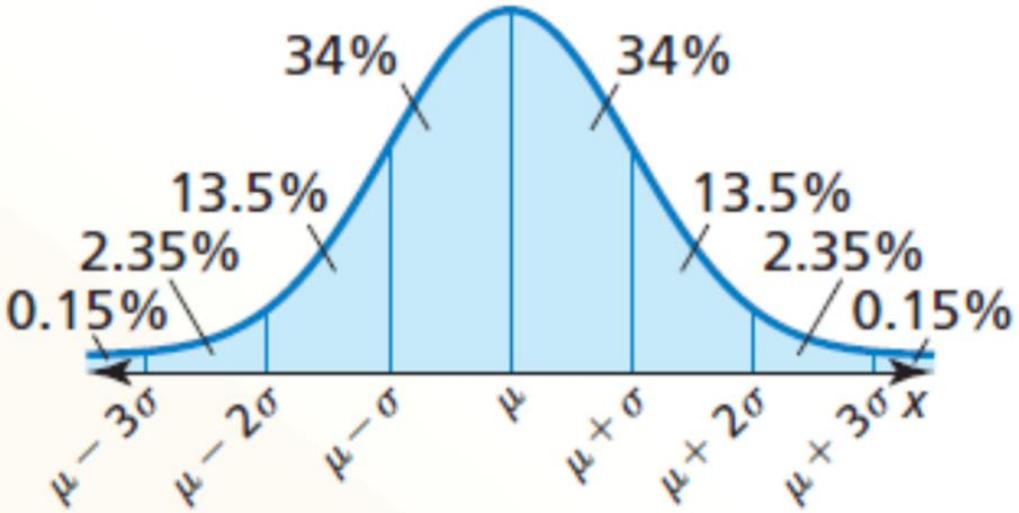
The scores for a state's peace officer standards and training test are normally distributed with a mean of 55 and a standard deviation of 12. The test scores range from 0 to 100.

a. About what percent of the people taking the test have scores between 43 and 67?



b. An agency in the state will only hire applicants with test scores of 67 or greater. About what percent of the people have test scores that make them eligible to be hired by the agency?





A normal distribution has mean μ and standard deviation σ. Find the indicated probability for a randomly selected *x*-value from the distribution.

1. $P(x \leq \mu)$

2. $P(x \geq \mu)$

3. $P(\mu \leq x \leq \mu + 2\sigma)$

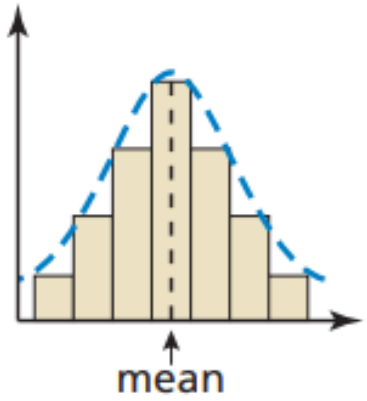
4. $P(\mu - \sigma \leq x \leq \mu)$

UNDERSTANDING MATHEMATICAL TERMS

Be sure you understand that you cannot use a normal distribution to interpret skewed distributions. The areas under a normal curve do not correspond to the areas of a skewed distribution.

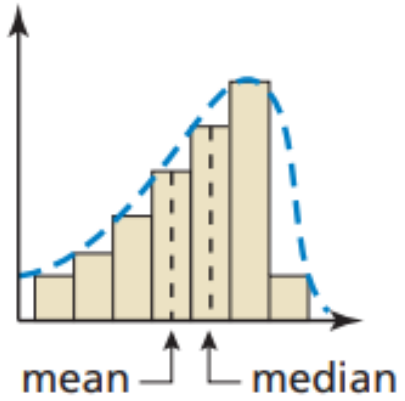
Recognizing Normal Distributions

Not all distributions are normal. For instance, consider the histograms shown below. The first histogram has a normal distribution. Notice that it is bell-shaped and symmetric. Recall that a distribution is symmetric when you can draw a vertical line that divides the histogram into two parts that are mirror images. Some distributions are skewed. The second histogram is *skewed left* and the third histogram is *skewed right*. The second and third histograms do *not* have normal distributions.



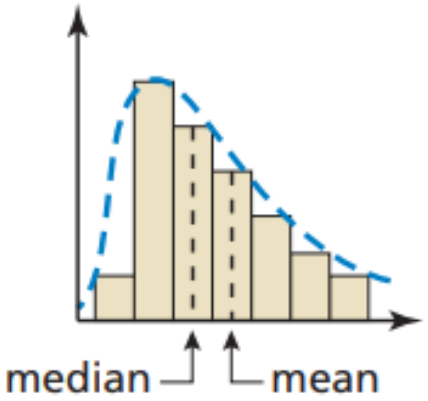
Bell-shaped and symmetric

- histogram has a normal distribution
- mean = median



Skewed left

- histogram does not have a normal distribution
- mean < median

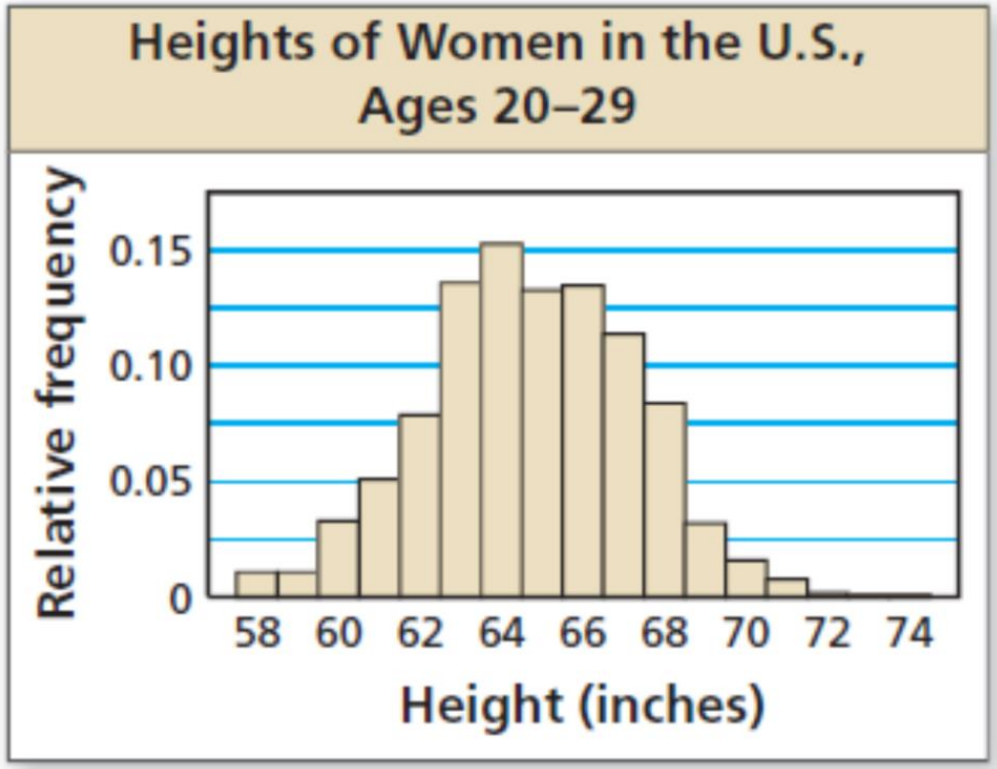


Skewed right

- histogram does not have a normal distribution
- mean > median

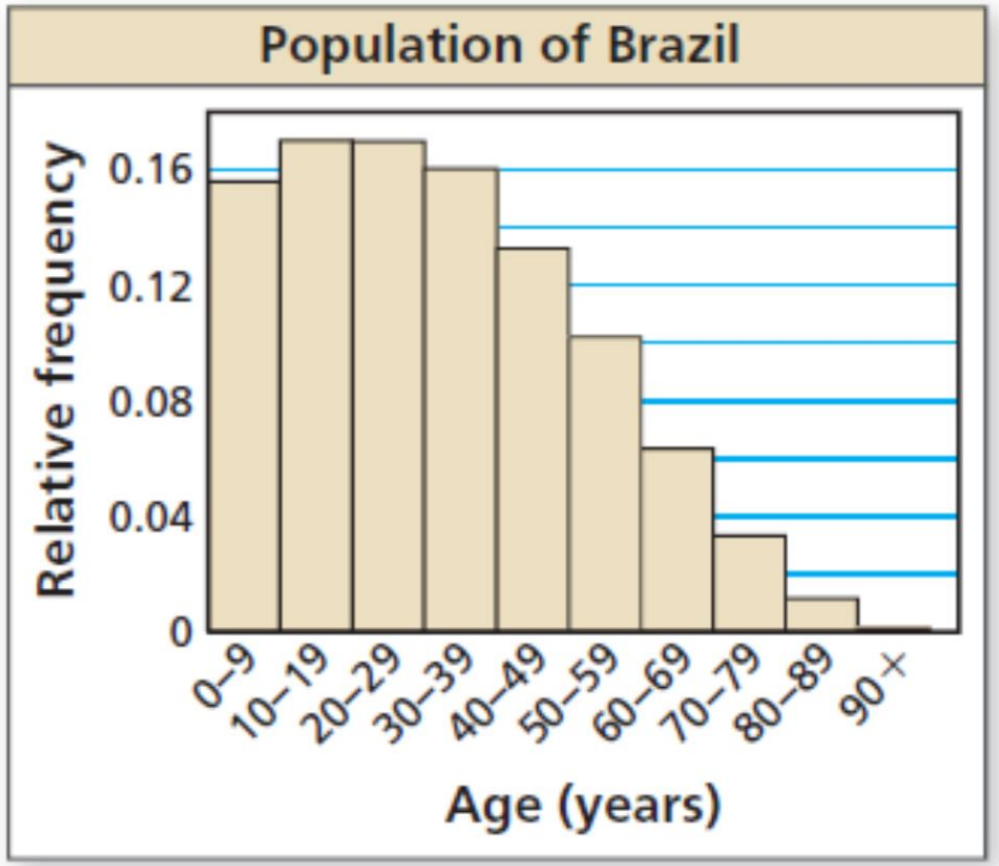
Determine whether each histogram has a normal distribution.

a.



The histogram is bell-shaped and fairly symmetric. So, the histogram has an approximately normal distribution.

b.



The histogram is skewed right. So, the histogram does not have a normal distribution, and you cannot use a normal distribution to interpret the histogram.

Homework: Textbook page 600; 3 – 17 odd, 19, 20